

Modified Enlarged 24pt
OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Monday 23 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y436/01 Further Pure with Technology

Time allowed: 1 hour 45 minutes
plus your additional time allowance

YOU MUST HAVE:

the Printed Answer Booklet or any
suitable paper provided by the centre. The
Printed Answer Booklet may be enlarged
by the centre

the Formulae Booklet for Further
Mathematics B (MEI)

a computer with appropriate software

a scientific or graphical calculator

Insert for Questions 1(a)(i), 1(f), 3(a)(iii),
3(b) (with this document)

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 60.

The marks for each question are shown in brackets [].

ADVICE

Read each question carefully before you start your answer.

- 1 A family of curves is given by the cartesian equation**

$$\frac{x^2}{a^2} + mxy + \frac{y^2}{b^2} = 1$$

where a , b and m are real numbers with a and b non-zero.

- (a) In this part of the question $a = 2$ and $b = 1$.**

- (i) On the axes in the Printed Answer Booklet or in the insert, sketch the curve in each of these cases.**

- $m = 0$**
- $m = 1$**
- $m = 2$ [3]**

- (ii) State ONE feature of the curve for the case $m = 0$ that is NOT a feature of the curve in the cases $m = 1$ and $m = 2$. [1]**

For the remainder of this question $m = 0$.

(b) Verify that the parametric equations of the curve are

$$x(t) = a \cos(t), y(t) = b \sin(t),$$

where $0 \leq t < 2\pi$ is a parameter. [1]

(c) Show that $\frac{dy}{dx} = -\frac{b}{a} \cot(t)$. [2]

(d) Show that the equation of the normal to the curve at the point with parameter t is

$$y = \frac{a}{b} \tan(t)x - \left(\frac{a^2 - b^2}{b} \right) \sin(t). \quad [5]$$

- (e) Show that the parametric equations of the envelope of the normal to the curve are

$$x(t) = \left(\frac{a^2 - b^2}{a} \right) \cos^3(t),$$

$$y(t) = \left(\frac{b^2 - a^2}{b} \right) \sin^3(t),$$

where $0 \leq t < 2\pi$ is a parameter. [6]

- (f) In this part of the question $a = 2$ and $b = 1$.

On the axes in the Printed Answer Booklet or in the insert, sketch the envelope of the normal to the curve. [1]

- (g) By considering the expression $(ax(t))^{\frac{2}{3}} + (by(t))^{\frac{2}{3}}$ or otherwise, determine a cartesian equation of the envelope of the normal to the curve. [2]

2 (a) (i) Write down $7^{10} \pmod{1000}$. [1]

Fermat's little theorem states that if p is a prime and x is an integer which is co-prime to p , then $x^{p-1} \equiv 1 \pmod{p}$.

(ii) Explain why Fermat's little theorem implies $2^{12} \equiv 1 \pmod{13}$. [1]

(iii) Determine $2^{(12q+1)} \pmod{13}$, where q is a positive integer. [2]

(b) In the rest of this question the highest common factor of positive integers m and n is denoted by (m, n) .

(i) Write down $(354, 27)$. [1]

Euler's totient function $\varphi(n)$, where n is a positive integer, is defined to be the number of integers m with $1 \leq m \leq n$ such that $(m, n) = 1$.

For example, $\varphi(12) = 4$ since 1, 5, 7 and 11 are all co-prime with 12, but 2, 3, 4, 6, 8, 9, 10, and 12 all share a common factor greater than 1 with 12.

(ii) Create a program which returns the value of $\varphi(n)$ for a given positive integer n .

Write out your program in full in the Printed Answer Booklet or the paper provided. [4]

(iii) Use your program to find $\varphi(1000)$. [1]

Euler's theorem states that if a and n are co-prime positive integers, then $a^{\varphi(n)} \equiv 1 \pmod{n}$.

(iv) Determine $7^{(400r+10)} \pmod{1000}$, where r is a positive integer. [4]

(v) Using part (b)(iv), determine the tens digit of 7^{2010} . [2]

(c) Suppose that $p \geq 3$ is a prime number and that x and y are positive integers.

By considering the equation $p^{2x} + 1 = 2^{2y}$ modulo 4, or otherwise, prove there are NO integer solutions to the equation $p^{2x} + 1 = 2^{2y}$. [5]

3 This question concerns the family of differential equations

$$\frac{dy}{dx} = x - y + ae^x \sin(y) \quad (**)$$

where a is a constant.

(a) In this part of the question $a = 0$.

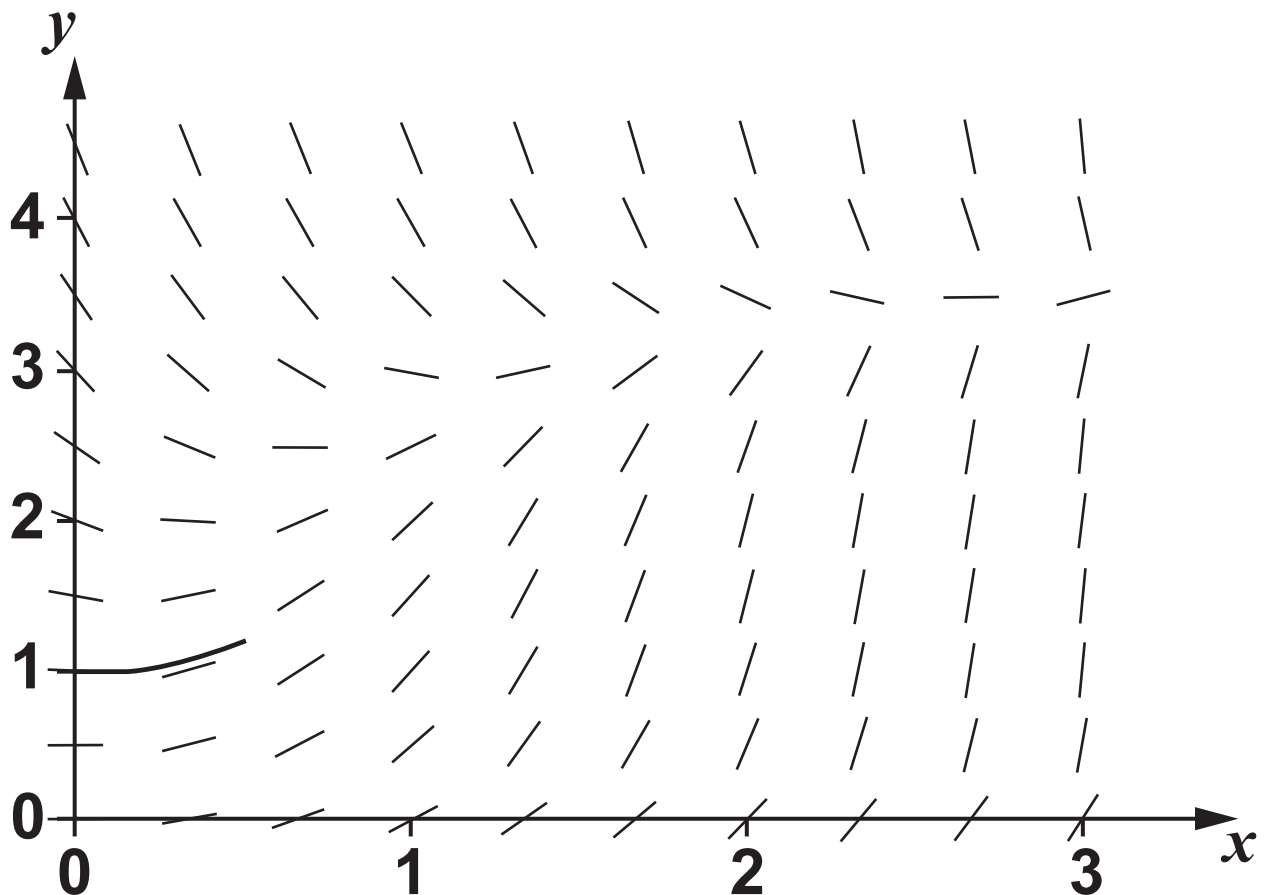
(i) Verify that $y = 2e^{-x} + x - 1$ is the particular solution of () that satisfies $y = 1$ when $x = 0$. [3]**

The solution $y = 2e^{-x} + x - 1$ has a minimum value at the point (m, n) where $0 < m < 1$.

(ii) Find the exact value of m . [2]

(iii) Sketch the particular solution of () given in part (a)(i) for $0 \leq x \leq 3$ on the axes in the Printed Answer Booklet or in the insert. [2]**

(b) The figure below shows the tangent field for an unspecified value of a . A sketch of the solution curve $y = g(x)$ which passes through the point $(0, 1)$ is shown for $0 \leq x \leq \frac{1}{2}$.



Continue the sketch of the solution curve for $\frac{1}{2} \leq x \leq 3$ on the axes in the PRINTED ANSWER BOOKLET or in the insert. [2]

- (c) (i) The standard Runge-Kutta method of order 4 for the solution of the differential equation $\frac{dy}{dx} = f(x, y)$ is as follows.

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Construct a spreadsheet to solve () so that the value of a and the value of h can be varied in the case $x_0 = 0$ and $y_0 = 4$. State the formulae you have used in your spreadsheet. [4]**

- (ii) In this part of the question $a = 1$, $x_0 = 0$ and $y_0 = 4$.

Use your spreadsheet with $h = 0.05$ to determine an approximation of the value of y for the solution to (**) when $x = 1$, giving your answer correct to 2 decimal places. [1]

- (iii) In this part of the question $a = 0.5$, $x_0 = 0$ and $y_0 = 4$.

The solution to (**) has a minimum point (r, s) with $1 < r < 2$.

Use your spreadsheet with suitable values of h to determine the value of r correct to 2 decimal places. [2]

(iv) In this part of the question
 $h = 0.05$, $x_0 = 0$ and $y_0 = 4$.

There is a value of $a > 0$
such that $y(1)$ is 2.47 to
2 decimal places.

By varying the value of a
in your spreadsheet, find a
suitable value of a correct to
1 decimal place. [2]

END OF QUESTION PAPER

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